education equals

Topic: Euler's rule, constructing networks, shortest path

Time: 45 mins   Marks: /45 marks

Calculator Assumed

Question One: [3, 2, 4, 2, 1: 12 marks]

a)   Construct a traversable network with one closed path/cycle and two odd nodes. Label your odd nodes and describe your closed cycle.

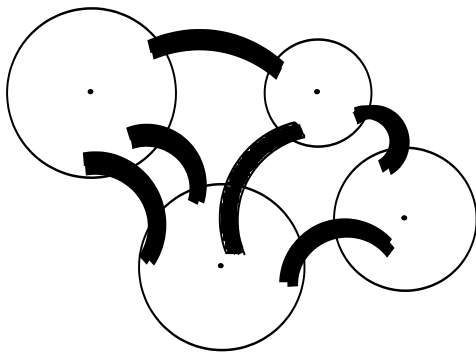b)   Construct a network which includes the trail ABCDE and closed walk ACDA.

c)   Consider the zones in the diagram below. Is it possible to draw a traversable network through all 13 "walls"? Justify your solution mathematically.
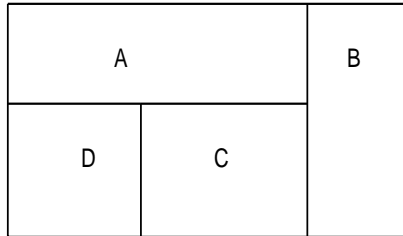

d)   Is it possible to construct a traversable network through the following diagram of islands and bridges?

If your answer is no, justify your solution and add one extra bridge which would make the network traversable. If your answer is yes, clearly show the traversable path.

Question Two: [2, 3, 5: 10 marks]

a)   The following is a map of Western Australia which has been divided into different
   agricultural regions.
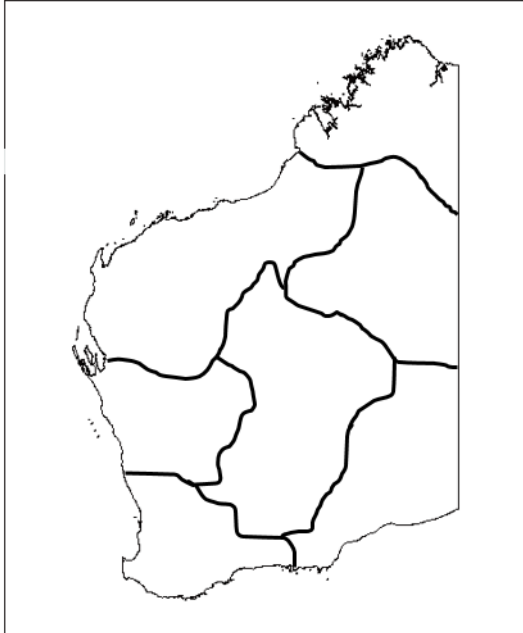
      i)   Adjacent to the map above, create a network to represent the map by
             representing the regions as hollow circles and the boundaries as arcs
             connecting these points.

      ii)   Use the network to determine and to demonstrate the minimum number of
             colours required to colour this map if regions (hollow circles) with a common
             boundary must be different colours.

b)   Create a network to show the minimum number of colours required to colour the
      map of Australia below if regions with a common boundary must be different colours.
   State the minimum number of colours required.
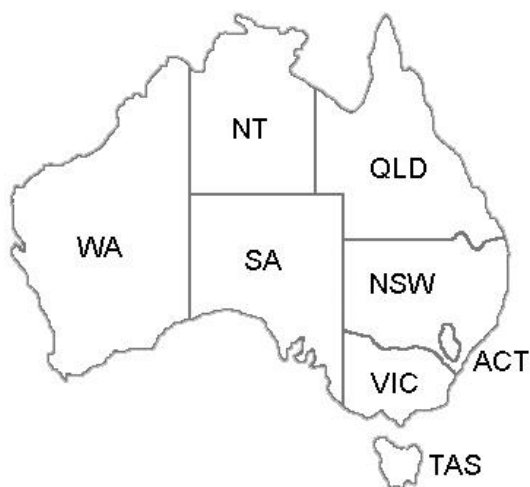
Question Three: [2, 3: 5 marks]

The following network shows the possible routes from Selena's house to her studio.
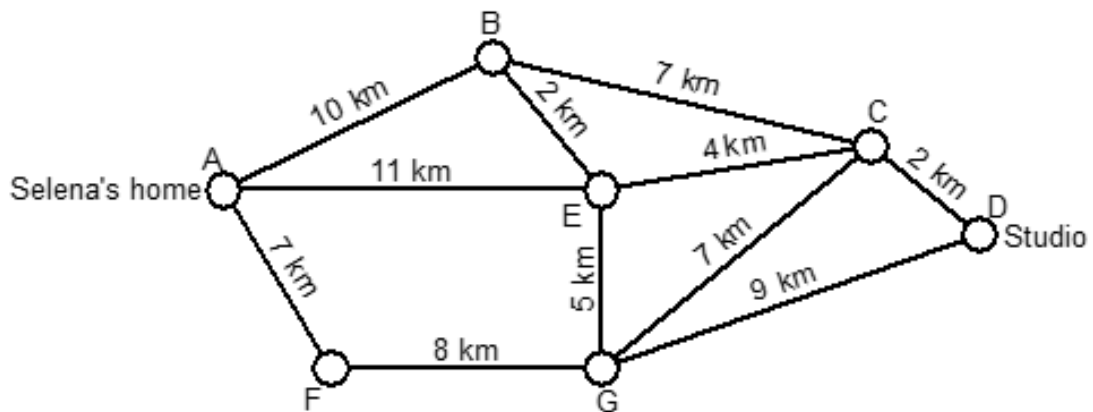
a)   State the shortest path and it's distance from Selena's house to the studio.

An accident at intersection C caused traffic to be diverted. Traffic is rerouted 2 km in on the path BC (at point H) and is taken to the middle of path CD (to point I). There is no alternative for traffic travelling from E or G to C.
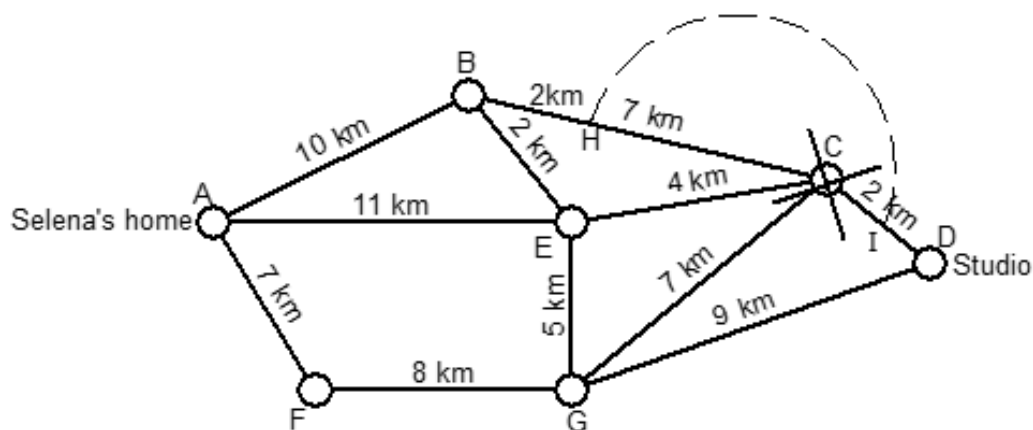

Selena is aware of the accident and the diversion before she leaves her home for the studio.

b)   If the diversion causes 2 km to be added to the previous shortest path from Selena's home to the studio how long is Path HI?

Question Four: [3, 1, 6: 10 marks]

The following network shows some of the corridors running through a large hospital. Most of the thoroughfare through this hospital is from A to E.
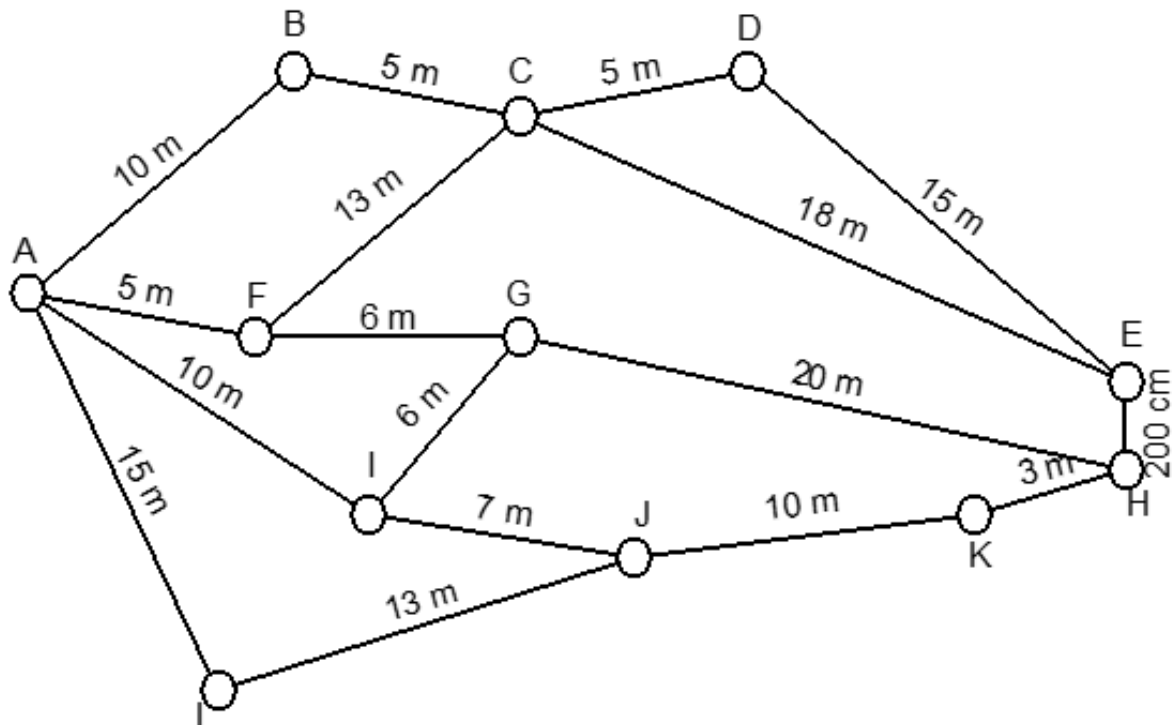
The hospital receives some funding to upgrade some of the finishings in the corridors. In order to maximise their budget they decided to upgrade the shortest path from A to E as these would be the most used corridors. The cost of upgrading is $95/metre.

a)   What is the shortest distance from A to E and what is this pathway?

b)   What is the cost of upgrading the shortest path from A to E?

Upon further discussions it appears that not all the corridors require the same amount of work to upgrade and therefore the costs of upgrading each corridor are not equal. The following table shows the multiplication factor in the costs of upgrading each corridor relative to the lengths.

	A	B	C	D	E	F	G	H	I	J	K	L
A		1				1.9			0.3			0.1
B	1		1.2									
C		1.2		0.9	1	0.2						
D			0.9		1.1							
E			1	1.1								
F	1.9		0.2				1.6	1				
G						1.6		2	0.1			
H					1		2				0.2	
I	0.3						0.5			0.3		
J									0.3		0.6	0.1
K								0.2		0.6		
L	0.1									0.1		

c)   Taking into account the multiplication factors, which path from A to E should the hospital upgrade in order to minimise costs? Clearly state this path and the total cost of upgrading it. Use the blank network below to assist with your answer.
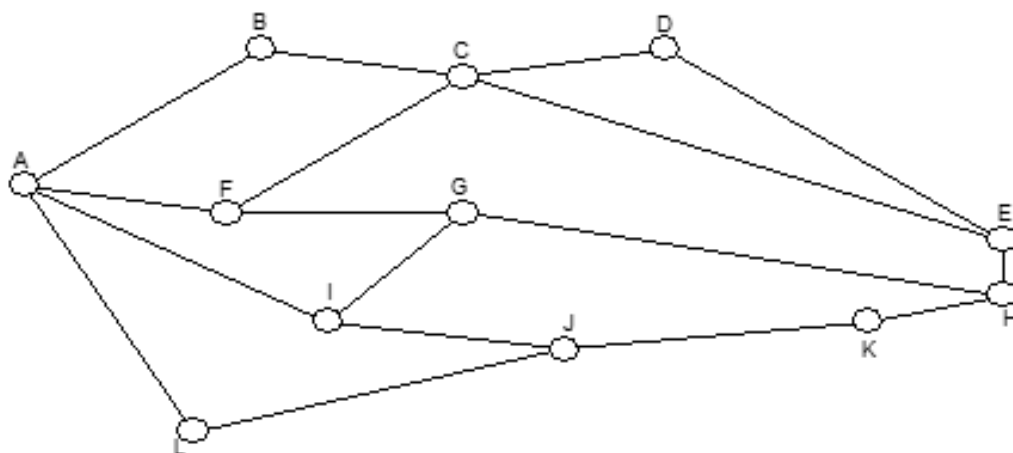
Question Five: [8 marks]
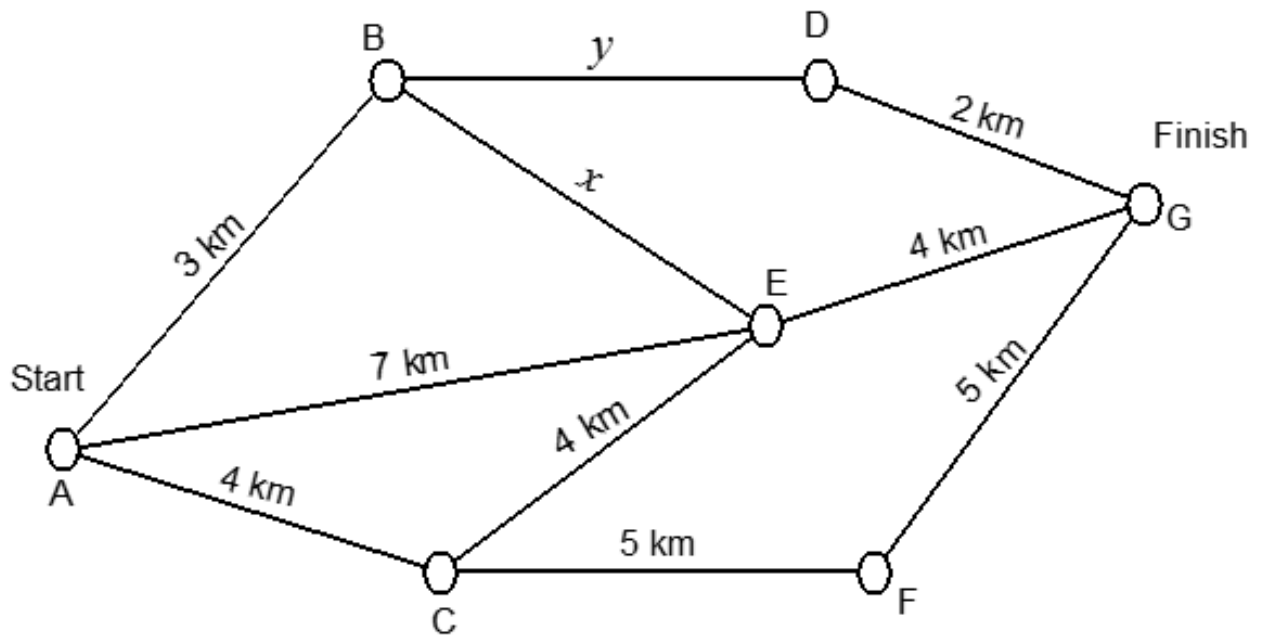
If the shortest path from start to finish is 9 km, state all possible values for x and y and the possible shortest path.

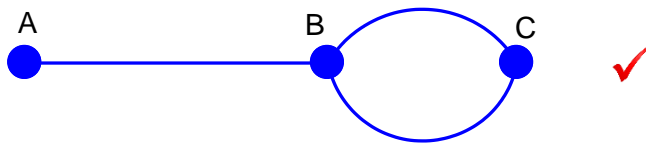**Topic: Euler's rule, constructing networks SOLUTIONS**

Time: 45 mins   Marks: /45 marks

Calculator Assumed

Question One: [3, 2, 4, 2, 1: 12 marks]

a)   Construct a traversable network with one closed path/cycle and two odd nodes. Label your odd nodes and describe your closed cycle.
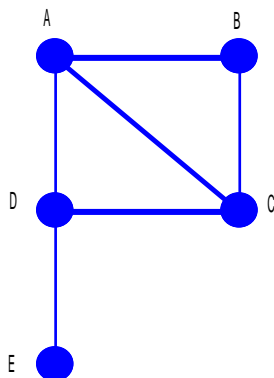

BCB is the closed cycle and nodes A and B are the two odd nodes.
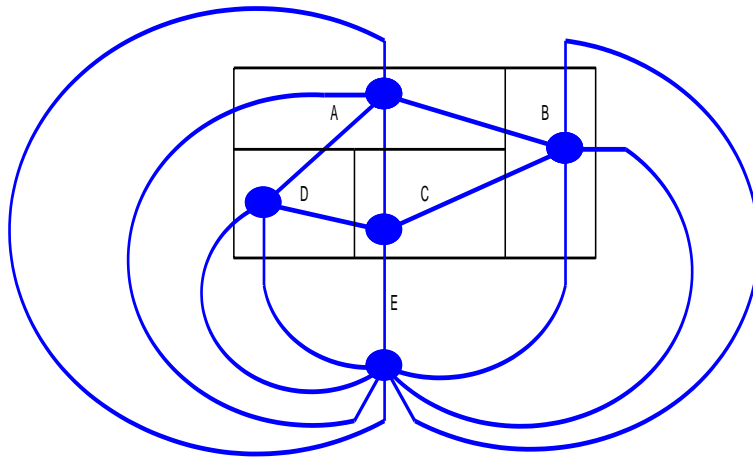
(variations in answers will also be correct)

b)   Construct a network which includes the trail ABCDE and closed walk ACDA.


(variations in answers will also be correct)

c) Consider the zones in diagram below. Is it possible to draw a traversable network through all 13 "walls"? Justify your solution mathematically.

Yes traversable as only two odd nodes.

d) Is it possible to construct a traversable network through the following diagram of islands and bridges?

If your answer is no, justify your solution and add one extra bridge which would make the network traversable. If your answer is yes, clearly show the traversable path.
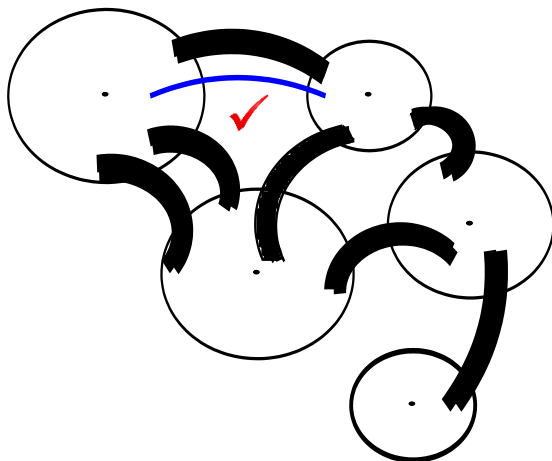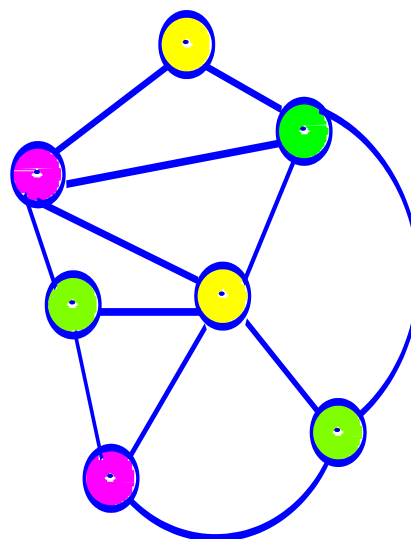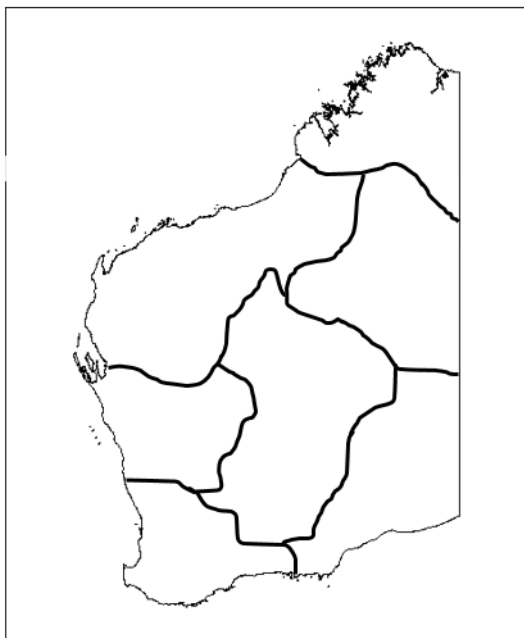

No, since there are 4 odd nodes.

Any path between two odd nodes will result in a correct answer.

Question Two: [2, 3, 5: 10 marks]

a)   The following is a map of Western Australia which has been divided into different agricultural regions.

   i)   Adjacent to the map above, create a network to represent it by representing the regions as hollow circles and the boundaries as arcs connecting these points. ✔ ✔

   ii)   Use the network to determine and to demonstrate the minimum number of colours required to colour this map if regions with a common boundary must be different colours. Minimum of 3 colours. ✔ ✔ ✔
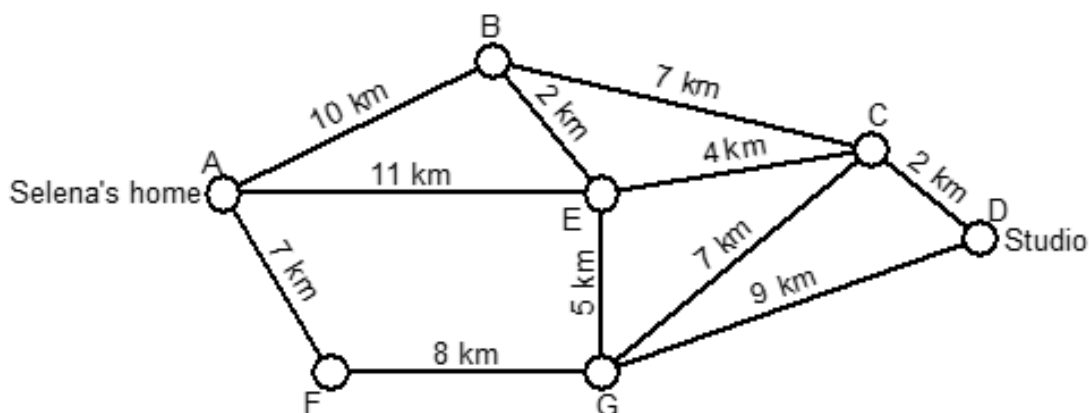
b)   Create a network to show the minimum number of colours required to colour the map of Australia below if regions with a common boundary must be different colours. State the minimum number of colours required.
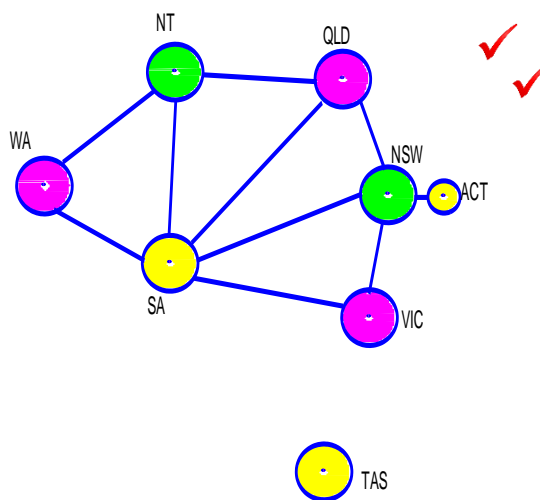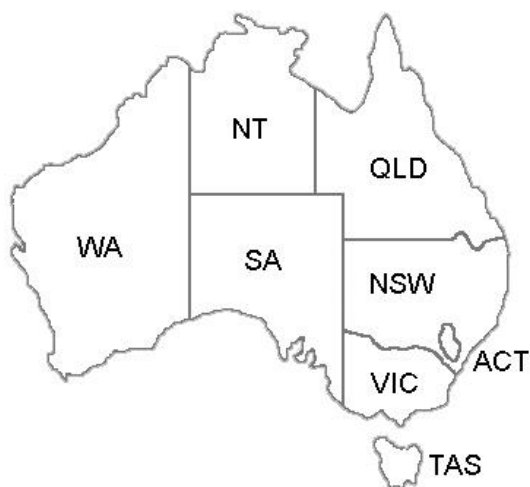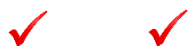

Minimum of 3 colours required. ✔ ✔ ✔

Question Three: [2, 3: 5 marks]

The following network shows the possible routes from Selena's house to her studio.
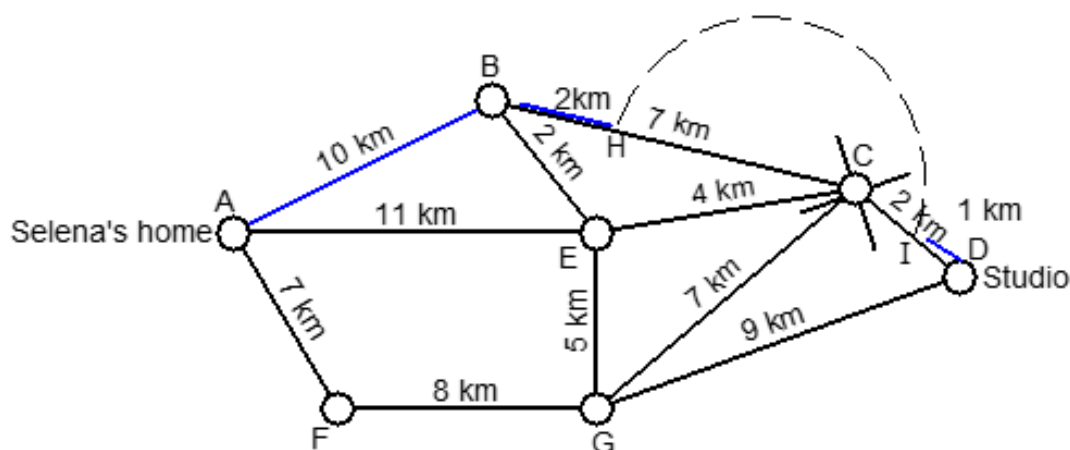

a)   State the shortest path and it's distance from Selena's house to the studio.

17 km   A-E-C-D

✔   ✔

An accident at intersection C caused traffic to be diverted. Traffic is rerouted 2 km in on the path BC (at point H) and is taken to the middle of path CD (to point I). There is no alternative for traffic travelling from E or G to C.

Selena is aware of the accident and the diversion before she leaves her home for the studio.
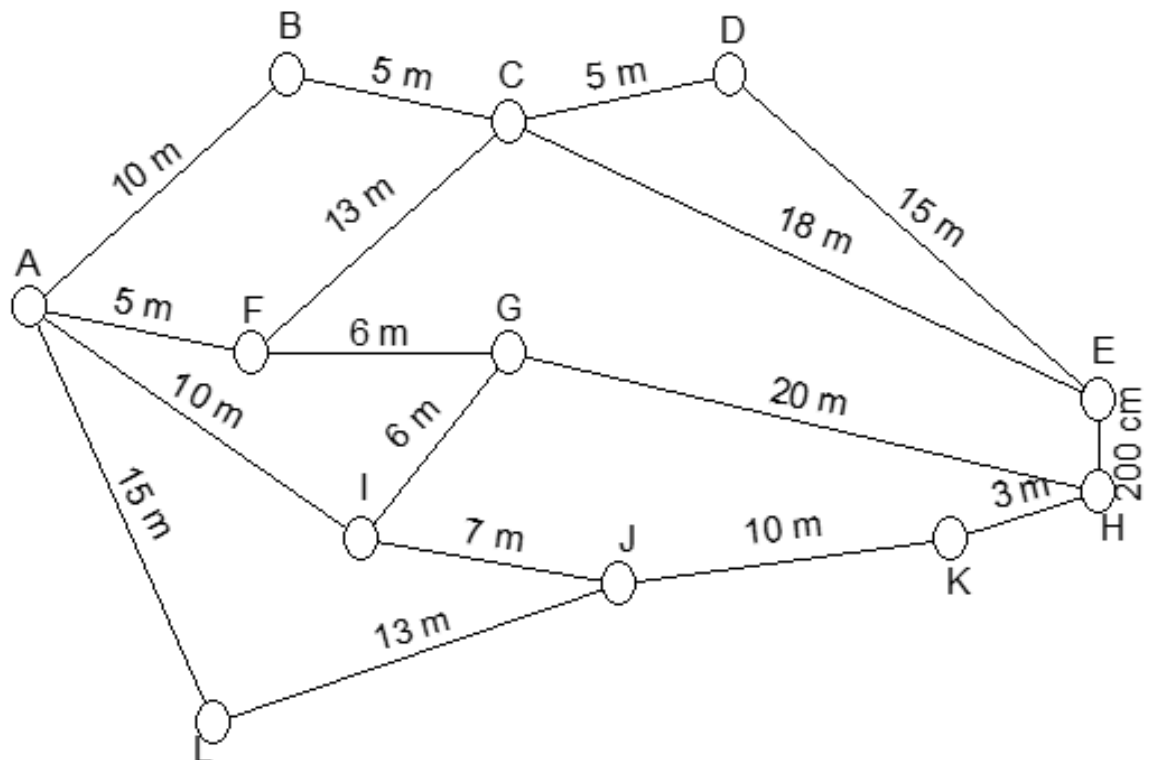
b)   If the diversion causes 2 km to be added to the previous shortest path from Selena's home to the studio how long is Path HI?

✔   ✔   ✔

$13 + 6 = 19\ km\ \therefore diversion\ is\ 6\ km\ long$

Question Four: [3, 1, 6: 10 marks]

The following network shows some of the corridors running through a large hospital. Most of the thoroughfare through this hospital is from A to E.

The hospital receives some funding to upgrade some of the finishing in the corridors. In order to maximise their budget they decided to upgrade the shortest path from A to E as these would be the most used corridors. The cost of upgrading is $95/metre.

a)   What is the shortest distance from A to E and what is this pathway?

A-I-J-K-H-E 32 km

✓ ✓   ✓

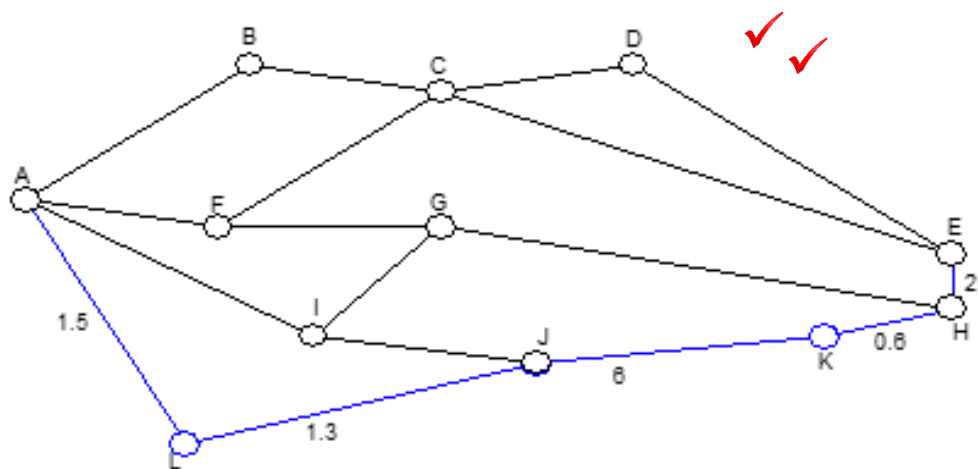b)   What is the cost of upgrading the shortest path from A to E?

$$32 \times 95 = \$3040 \quad ✓$$

Upon further discussions it appears that not all the corridors require the same amount of work to upgrade and therefore the costs of upgrading each corridor are not equal. The following table shows the multiplication factor in the costs of upgrading each corridor relative to the lengths.

	A	B	C	D	E	F	G	H	I	J	K	L
A		1				1.9			0.3			0.1
B	1		1.2									
C		1.2		0.9	1	0.2						
D			0.9		1.1							
E			1	1.1								
F	1.9		0.2				1.6	1				
G						1.6		2	0.1			
H					1		2				0.2	
I	0.3						0.5			0.3		
J									0.3		0.6	0.1
K								0.2		0.6		
L	0.1									0.1		

c) Taking into account the multiplication factor, which path from A to E should the hospital upgrade in order to minimise costs? Clearly state this path and the total cost of upgrading it. Use the blank network below to assist with your answer.
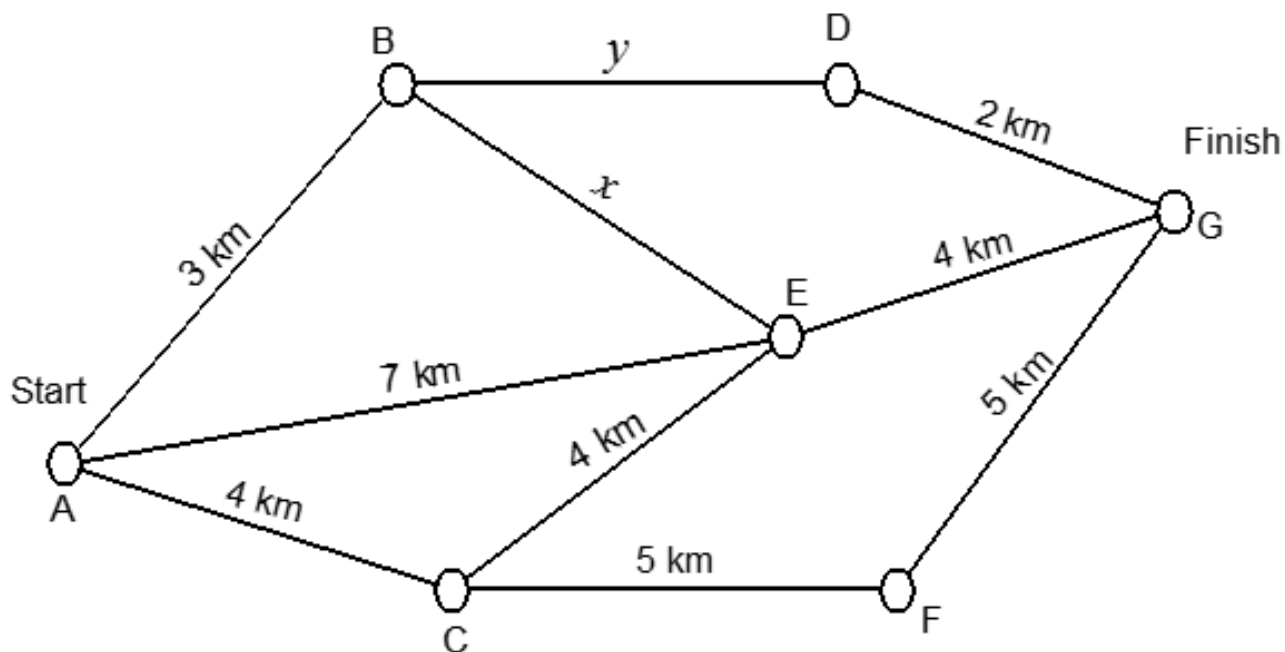
The cheapest path to upgrade would be A-L-J-K-H-E. ✓ ✓

$$1.5 + 1.3 + 6 + 0.6 + 0.5 = 9.9$$ ✓

$$9.9 \times 95 = \$940.50$$ ✓

Total cost of upgrading this path is $940.50

Question Five: [8 marks]

If the shortest path from start to finish is 9 km, state all possible values for x and y and the possible shortest path.

$3 + y + 2 = 9$

$y = 4$

$Path: A - B - D - G$

$y = 4 \quad and \quad x > 2$

OR

$3 + x + 4 = 9$ ✓

$x = 2$ ✓

$Path: A - B - E - G$ ✓

$x = 2 \quad and \quad y > 4$ ✓

✓

✓

✓

✓